Topics 1 & 2
Algebra and Quadratics

Bronze, Silver, Gold and Platinum Worksheets for AS level Mathematics

Teacher Notes

These Bronze, Silver and Gold worksheets are designed to be used either straight after the content has been taught or as part of a skills gap analysis, especially as students move into year 13.

They are drawn from the latest specification questions and legacy questions. The papers are between 25 and 35 marks.

The topic number on this worksheet relates to the corresponding chapter number in the 'Pearson Edexcel AS and A Level Mathematics: Pure Mathematics Year 1/AS' textbook.

Non-Calculator Questions

The new specification allows calculators to be used in all papers. We have, however, put these questions together with the intention that students can complete them without a calculator. It's important for pupils to be able to maintain their non-calculator skills, especially on topics such as surds or indices, to support question that use the keywords "show that" or "prove". If you wish to ease the difficulty slightly then you can, of course, allow students to attempt them with the support of a calculator.

Quick Links

(Press Ctrl, as you click with your mouse to follow these links)

- Bronze Questions
- Bronze Mark Scheme
- Silver Questions
- Silver Mark Scheme
- Gold Questions
- Gold Mark Scheme

The Platinum Questions below are taken from the Advanced Extension Award. You can use these in class as high level problem solving questions, either with individual students or as group problem solving exercises. On the Advanced Extension Award students, typically, need to get around 50% to get a Merit and around 70% to get a distinction.

- Platinum Questions
- Platinum Mark Schemes

Extension and Enrichment

If you have students that have enjoyed the challenge of the Gold questions, then they should have a go at the more challenging question from our Advanced Extension Award (AEA) papers. The Mathematics AEA is a single, 3 hour non-calculator paper, taken at the end of year 13. It helps students to develop high level problem solving and proof skills. It is entirely based on the content of the A Level Mathematics Course. No extra material needs to be covered to take the AEA in Mathematics. A second important difference is that marks are awarded for the clarity and quality of their solution. Developing this key skill, alongside the extra problem-solving experience, can pay dividends in the way they approach A Level Mathematics and Further Mathematics problems.

More information about the Advanced Extension Award can be found here on the Pearson Edexcel Website, or here on the Maths Emporium

Bronze Questions

Calculators may not be used

The total mark for this section is 32

Q1

(a) Find the value of $8^{\frac{4}{3}}$

(2)

(b) Simplify $\dfrac{15x^{\frac{4}{3}}}{3x}$

(2)

(Total for Question 1 is 4 marks)

Q2

(a) Write down the value of $32^{\frac{1}{5}}$

(1)

(b) Simplify fully $(32x^5)^{-\frac{2}{5}}$

(3)

(Total for Question 2 is 4 marks)

Q3

(a) Simplify

$$\sqrt{50} - \sqrt{18}$$

giving your answer in the form $a\sqrt{2}$, where a is an integer.

(2)

(b) Hence, or otherwise, simplify

$$\frac{12\sqrt{3}}{\sqrt{50} - \sqrt{18}}$$

giving your answer in the form $b\sqrt{c}$, where b and c are integers and $b \neq 1$

(3)

(Total for Question 3 is 5 marks)

Q4

Given that $32\sqrt{2} = 2^a$, find the value of a

(Total for Question 4 is 3 marks)

Q5

(a) Evaluate $32^{\frac{3}{5}}$, giving your answer as an integer.

(2)

(b) Simplify fully $\left(\dfrac{25x^4}{4}\right)^{-\frac{1}{2}}$

(2)

(Total for Question 5 is 4 marks)

Q6

Express 8^{2x+3} in the form 2^y, stating y in terms of x

(Total for Question 6 is 2 marks)

Q7

Complete the table below. The first one has been done for you.

For each statement you must state if it is always true, sometimes true or never true, giving a reason in each case.

Statement	Always True	Sometimes True	Never True	Reason
The quadratic equation $ax^2 + bx + c = 0, \quad (a \neq 0)$ has 2 real roots.		✓		It only has 2 real roots when $b^2 - 4ac > 0$. When $b^2 - 4ac = 0$ it has 1 real root and when $b^2 - 4ac < 0$ it has 0 real roots.
(i) When a real value of x is substituted into $x^2 - 6x + 10$ the result is positive. (2)				
(ii) If $ax > b$ then $x > \frac{b}{a}$ (2)				
(iii) The difference between consecutive square numbers is odd. (2)				

(Total for Question 7 is 6 marks)

Pearson

Q8

The equation $x^2 + 3px + p = 0$, where p is a non-zero constant, has equal roots.

Find the value of p.

(Total for Question 8 is 4 marks)

End of Questions

Bronze Mark Scheme

Q1

Question number	Scheme		Marks	
	(a) Attempt $\sqrt[3]{8}$ or $\sqrt[3]{(8^4)}$		M1	
	$= \underline{16}$		A1	(2)
	(b) $\underline{5x^{\frac{1}{3}}}$	$5,\ x^{\frac{1}{3}}$	B1, B1	(2)
				4

(a) M1 for: 2 (on its own) or $\left(2^3\right)^{\frac{4}{3}}$ or $\sqrt[3]{8}$ or $\left(\sqrt[3]{8}\right)^4$ or 2^4 or $\sqrt[3]{8^4}$ or $\sqrt[3]{4096}$

8^3 or 512 or $(4096)^{\frac{1}{3}}$ is M0

A1 for 16 only

(b) 1st B1 for 5 on its own or $\times$ something.

So e.g. $\dfrac{5x^{\frac{4}{3}}}{x}$ is B1 But $5^{\frac{1}{3}}$ is B0

An expression showing cancelling is not sufficient

(see first expression of QC0184500123945 the mark is scored for the second expression)

2nd B1 for $x^{\frac{1}{3}}$

Can use ISW (incorrect subsequent working)

e.g $5x^{\frac{4}{3}}$ scores B1B0 but it may lead to $\sqrt[3]{5x^4}$ which we ignore as ISW.

Correct answers only score full marks in both parts.

Q2

Question Number	Scheme	Marks
	(a) $32^{\frac{1}{5}} = 2$	B1 (1)
	(b) For 2^{-2} or $\frac{1}{4}$ or $\left(\frac{1}{2}\right)^2$ or 0.25 as coefficient of x^k, for any value of k including $k = 0$	M1
	Correct index for x so Ax^{-2} or $\frac{A}{x^2}$ o.e. for any value of A	B1
	$= \frac{1}{4x^2}$ or $0.25x^{-2}$	A1 cao (3)
		4 Marks

Notes

(a) B1 Answer 2 must be in part (a) for this mark

(b) Look at their final answer

M1 For 2^{-2} or $\frac{1}{4}$ or $\left(\frac{1}{2}\right)^2$ or 0.25 in their answer as coefficient of x^k for numerical value of k (including $k = 0$) so final answer $\frac{1}{4}$ is M1 B0 A0

B1 Ax^{-2} or $\frac{A}{x^2}$ or equivalent e.g. $Ax^{-\frac{10}{5}}$ or $Ax^{-\frac{50}{25}}$ i.e. correct power of x seen in final answer

May have a bracket provided it is $(Ax)^{-2}$ or $\left(\frac{A}{x}\right)^2$

A1 $\frac{1}{4x^2}$ or $\frac{1}{4}x^{-2}$ or $0.25\,x^{-2}$ oe but must be correct power **and** coefficient combined correctly and must not be followed by a different wrong answer.

Poor bracketing: $2x^{-2}$ earns M0 B1 A0 as correct power of x is seen in this solution (They can recover if they follow this with $\frac{1}{4x^2}$ etc)

Special case $(2x)^{-2}$ as a **final** answer and $\left(\frac{1}{2x}\right)^2$ can have M0 B1 A0 if the correct expanded answer is not seen

The correct answer $\frac{1}{4x^2}$ etc. followed by $\left(\frac{1}{2x}\right)^2$ or $(2x)^{-2}$, treat $\frac{1}{4x^2}$ as final answer so M1 B1 A1 isw

But the correct answer $\frac{1}{4x^2}$ etc clearly followed by the wrong $2x^{-2}$ or $4x^{-2}$, gets M1 B1 A0 do not ignore subsequent wrong work here

Q3

Question Number	Scheme	Notes	Marks
(a)	$\sqrt{50}-\sqrt{18}=5\sqrt{2}-3\sqrt{2}$	$\sqrt{50}=5\sqrt{2}$ or $\sqrt{18}=3\sqrt{2}$ **and the other term** in the form $k\sqrt{2}$. This mark may be implied by the correct answer $2\sqrt{2}$	M1
	$=2\sqrt{2}$	Or $a=2$	A1
			[2]
(b) WAY 1	$\dfrac{12\sqrt{3}}{\sqrt{50}-\sqrt{18}}=\dfrac{12\sqrt{3}}{"2"\sqrt{2}}$	Uses part (a) by replacing denominator by their $a\sqrt{2}$ where a is numeric. This is all that is required for this mark.	M1
	$=\dfrac{12\sqrt{3}}{"2"\sqrt{2}}\times\dfrac{\sqrt{2}}{\sqrt{2}}=\dfrac{12\sqrt{6}}{4}$	Rationalises the denominator by a correct method e.g. multiplies numerator and denominator by $k\sqrt{2}$ to obtain a multiple of $\sqrt{6}$. Note that multiplying numerator and denominator by $2\sqrt{2}$ or $-2\sqrt{2}$ is quite common and is acceptable for this mark. May be implied by a correct answer. **This is dependent on the first M1.**	dM1
	$=3\sqrt{6}$ or $b=3,\ c=6$	Cao and cso	A1
			[3]
(b) WAY 2	$\dfrac{12\sqrt{3}}{\sqrt{50}-\sqrt{18}}\times\dfrac{\sqrt{50}+\sqrt{18}}{\sqrt{50}+\sqrt{18}}$ **or** $\dfrac{12\sqrt{3}}{5\sqrt{2}-3\sqrt{2}}\times\dfrac{5\sqrt{2}+3\sqrt{2}}{5\sqrt{2}+3\sqrt{2}}$	For rationalising the denominator by a correct method i.e. multiplying numerator and denominator by $k\left(\sqrt{50}+\sqrt{18}\right)$	M1
	$\dfrac{60\sqrt{6}+36\sqrt{6}}{50-18}$	For replacing numerator by $\alpha\sqrt{6}+\beta\sqrt{6}$. **This is dependent on the first M1 and there is no need to consider the denominator for this mark.**	dM1
	$=3\sqrt{6}$ or $b=3,\ c=6$	Cao and cso	A1
			[3]
(b) WAY 3	$\dfrac{12\sqrt{3}}{\sqrt{50}-\sqrt{18}}=\dfrac{12\sqrt{3}}{"2"\sqrt{2}}$	Uses part (a) by replacing denominator by their $a\sqrt{2}$ where a is numeric. This is all that is required for this mark.	M1
	$=\dfrac{12\sqrt{3}}{2\sqrt{2}}=\dfrac{6\sqrt{3}}{\sqrt{2}}=\dfrac{\sqrt{108}}{\sqrt{2}}=\sqrt{54}=\sqrt{9}\sqrt{6}$	Cancels to obtain a multiple of $\sqrt{6}$. **This is dependent on the first M1.**	dM1
	$=3\sqrt{6}$ Or $b=3,\ c=6$	Cao and cso	A1
			[3]
(b) WAY 4	$\dfrac{12\sqrt{3}}{\sqrt{50}-\sqrt{18}}=\dfrac{12\sqrt{3}}{"2"\sqrt{2}}$	Uses part (a) by replacing denominator by their $a\sqrt{2}$ where a is numeric. This is all that is required for this mark.	M1
	$\left(\dfrac{12\sqrt{3}}{"2"\sqrt{2}}\right)^2=\dfrac{432}{8}$		
	$\sqrt{54}=\sqrt{9}\sqrt{6}$	Obtains a multiple of $\sqrt{6}$. **This is dependent on the first M1.**	dM1
	$=3\sqrt{6}$ Or $b=3,\ c=6$	Cao and cso (do not allow $\pm3\sqrt{6}$)	A1
			5 marks

Q4

Question Number	Scheme	Marks
Q	$32 = 2^5$ or $2048 = 2^{11}$, $\sqrt{2} = 2^{1/2}$ or $\sqrt{2048} = (2048)^{\frac{1}{2}}$	B1, B1
	$a = \frac{11}{2}$ $\left(\text{or } 5\frac{1}{2} \text{ or } 5.5\right)$	B1
		[3]
	1st B1 for $32 = 2^5$ or $2048 = 2^{11}$ This should be explicitly seen: $32\sqrt{2} = 2^a$ followed by $2^5\sqrt{2} = 2^a$ is OK Even writing $32 \times 2 = 2^5 \times 2\left(= 2^6\right)$ is OK but simply writing $32 \times 2 = 2^6$ is NOT 2nd B1 for $2^{\frac{1}{2}}$ or $(2048)^{\frac{1}{2}}$ seen. This mark may be implied 3rd B1 for answer as written. **Need $a =$ …** so $2^{\frac{11}{2}}$ is B0 $a = \frac{11}{2}$ $\left(\text{or } 5\frac{1}{2} \text{ or } 5.5\right)$ with no working scores full marks. If $a = 5.5$ seen then award 3/3 unless it is clear that the value follows from totally incorrect work. Part solutions: e.g. $2^5\sqrt{2}$ scores the first B1. <u>Special case</u>: If $\sqrt{2} = 2^{1/2}$ is not explicitly seen, but the final answer includes $\frac{1}{2}$, e.g. $a = 2\frac{1}{2}$, $a = 4\frac{1}{2}$, the second B1 is given by implication.	

Q5

Question Number	Scheme		Marks
(a)	$\left\{(32)^{\frac{3}{5}}\right\} = \left(\sqrt[5]{32}\right)^3$ or $\sqrt[5]{(32)^3}$ or 2^3 or $\sqrt[5]{32768}$		M1
	$= 8$		A1
			[2]
(b)	$\left\{\left(\frac{25x^4}{4}\right)^{-\frac{1}{2}}\right\} = \left(\frac{4}{25x^4}\right)^{\frac{1}{2}}$ or $\left(\frac{5x^2}{2}\right)^{-1}$ or $\frac{1}{\left(\frac{25x^4}{4}\right)^{\frac{1}{2}}}$	*See notes below*	M1
	$= \frac{2}{5x^2}$ or $\frac{2}{5}x^{-2}$	*See notes for other alternatives*	A1
			[2] 4

	Notes
(a)	**M1**: for a full correct interpretation of the fractional power. **Note:** $5\times(32)^3$ is M0. **A1:** for 8 only. **Note:** Award M1A1 for writing down 8.
(b)	**M1:** For use of $\frac{1}{2}$ OR use of -1 Use of $\frac{1}{2}$: Candidate needs to demonstrate the they have rooted all three elements in their bracket. Use of -1: Either Candidate has $\frac{1}{\text{Bracket}}$ or $\left(\frac{Ax^C}{B}\right)$ becomes $\left(\frac{B}{Ax^C}\right)$. Allow M1 for... • $\left(\frac{4}{25x^4}\right)^{\frac{1}{2}}$ or $\left(\frac{5x^2}{2}\right)^{-1}$ or $\frac{1}{\left(\frac{25x^4}{4}\right)^{\frac{1}{2}}}$ or $\sqrt{\left(\frac{4}{25x^4}\right)}$ or $\frac{1}{\sqrt{\left(\frac{25x^4}{4}\right)}}$ or $\left(\frac{\frac{1}{25x^4}}{\frac{1}{4}}\right)^{\frac{1}{2}}$ or $\frac{\frac{1}{5x^2}}{\frac{1}{2}}$ or $\frac{\frac{1}{5}x^{-2}}{\frac{1}{2}}$ or $-\left(\frac{5x^2}{2}\right)$ or $\left(\frac{-5x^{-2}}{-2}\right)$ or $-\left(\frac{5x^{-2}}{2}\right)$ or $\frac{5x^{-2}}{2}$ • $\left(\frac{4}{25x^4}\right)^K$ or $\left(\frac{5x^2}{2}\right)^C$ where K, C are any powers including 1. **A1:** for either $\frac{2}{5x^2}$ or $\frac{2}{5}x^{-2}$ or $0.4x^{-2}$ or $\frac{0.4}{x^2}$. **Note:** $\left(\sqrt{\left(\frac{25x^4}{4}\right)}\right)^{-1}$ is not enough work by itself for the method mark. **Note:** A final answer of $\frac{1}{\frac{5}{2}x^2}$ or $\frac{1}{2\frac{1}{2}x^2}$ or $\frac{1}{2.5x^2}$ is A0. **Note:** Also allow $\pm\frac{2}{5x^2}$ or $\pm\frac{2}{5}x^{-2}$ or $\pm 0.4x^{-2}$ or $\pm\frac{0.4}{x^2}$ for A1.

Q6

Question Number	Scheme	Marks
	$(8^{2x+3} = (2^3)^{2x+3}) = 2^{3(2x+3)}$ or 2^{ax+b} with $a = 6$ or $b = 9$	M1
	$= 2^{6x+9}$ *or* $= 2^{3(2x+3)}$ as **final** answer with no errors or $(y =)6x + 9$ or $3(2x + 3)$	A1
		[2]
		2 marks
	Notes	
	M1: Uses $8 = 2^3$, and **multiplies** powers $3(2x + 3)$. Does **not** add powers. (Just $8 = 2^3$ or $8^{\frac{1}{3}} = 2$ is M0) **A1:** Either 2^{6x+9} *or* $= 2^{3(2x+3)}$ or $(y =)6x + 9$ or $3(2x + 3)$	
	Note: Examples: 2^{6x+3} scores M1A0 : $8^{2x+3} = (2^3)^{2x+3} = 2^{3+2x+3}$ gets M0A0 **Special case:** : $= 2^{6x}\,2^9$ without seeing as single power M1A0 **Alternative method using logs:** $8^{2x+3} = 2^y \Rightarrow (2x+3)\log 8 = y\log 2 \Rightarrow y = \dfrac{(2x+3)\log 8}{\log 2}$ **So** $(y =)6x + 9$ or $3(2x + 3)$	 **M1** **A1 [2]**

Q7

Question	Scheme	Marks	AOs
(i)	$x^2-6x+10=(x-3)^2+1$	M1	2.1
	Deduces "always true" as $(x-3)^2\geqslant 0\Rightarrow\ (x-3)^2+1\geqslant 1$ and so is always positive	A1	2.2a
		(2)	
(ii)	For an explanation that it need not (always) be true This could be if $a<0$ then $ax>b\Rightarrow x<\frac{b}{a}$	M1	2.3
	States 'sometimes' and explains if $a>0$ then $ax>b\Rightarrow x>\frac{b}{a}$ if $a<0$ then $ax>b\Rightarrow x<\frac{b}{a}$	A1	2.4
		(2)	
(iii)	Difference $=(n+1)^2-n^2=2n+1$	M1	3.1a
	Deduces "Always true" as $2n+1$ = (even +1) = odd	A1	2.2a
		(2)	
		(6 marks)	

Notes:

(i)

M1: Attempts to complete the square or any other valid reason. Allow for a graph of $y=x^2-6x+10$ or an attempt to find the minimum by differentiation

A1: States always true with a valid reason for their method

(ii)

M1: For an explanation that it need not be true (sometimes). This could be if $a<0$ then $ax>b\Rightarrow x<\frac{b}{a}$ or simply $-3x>6\Rightarrow x<-2$

A1: Correct statement (sometimes true) and explanation

(iii)

M1: Sets up the proof algebraically.
For example by attempting $(n+1)^2-n^2=2n+1$ or $m^2-n^2=(m-n)(m+n)$ with $m=n+1$

A1: States always true with reason and proof
Accept a proof written in words. For example
If integers are consecutive, one is odd and one is even
When squared $\text{odd}\times\text{odd}=\text{odd}$ and $\text{even}\times\text{even}=\text{even}$
The difference between odd and even is always odd, hence always true
Score M1 for two of these lines and A1 for a good proof with all three lines or equivalent.

Q8

Question Number	Scheme	Marks
Q	b^2-4ac attempted, in terms of p. $(3p)^2-4p=0$ o.e. Attempt to solve for p e.g. $p(9p-4)=0$ Must potentially lead to $p=k,\ k\neq 0$ $p=\frac{4}{9}$ (Ignore $p=0$, if seen)	M1 A1 M1 A1cso **[4]**
	1st M1 for an attempt to substitute into b^2-4ac or $b^2=4ac$ with b or c correct Condone x's in one term only. This can be inside a square root as part of the quadratic formula for example. **Use of inequalities can score the M marks only** 1st A1 for any correct equation: $(3p)^2-4\times1\times p=0$ or better 2nd M1 for an attempt to factorize or solve their quadratic expression in p. Method must be sufficient to lead to their $p=\frac{4}{9}$. Accept factors or use of quadratic formula or $\left(p\pm\frac{2}{9}\right)^2=k^2$ (o.e. eg) $\left(3p\pm\frac{2}{3}\right)^2=k^2$ or equivalent work on <u>their</u> eqn. $9p^2=4p\Rightarrow\frac{9p^{\cancel{2}}}{\cancel{p}}=4$ which would lead to $9p=4$ is OK for this 2nd M1 ALT <u>Comparing coefficients</u> M1 for $(x+\alpha)^2=x^2+\alpha^2+2\alpha x$ and A1 for a correct equation eg $3p=2\sqrt{p}$ M1 for forming solving leading to $\sqrt{p}=\frac{2}{3}$ or better <u>Use of quadratic/discriminant formula (or any formula) Rule for awarding M mark</u> If the formula is quoted accept some correct substitution leading to a partially correct expression. If the formula is not quoted only award for a fully correct expression using their values.	

Silver Questions

Calculators may not be used

The total mark for this section is 35

Q1

(a) Find the value of $16^{\frac{1}{4}}$

(2)

(b) Simplify $x(2x^{-\frac{1}{4}})^4$

(2)

(Total for Question 1 is 4 marks)

Q2

Show that $\dfrac{2}{\sqrt{(12)}-\sqrt{(8)}}$ can be written in the form $\sqrt{a}-\sqrt{b}$, where a and b are integers.

(Total for Question 2 is 5 marks)

Q3

Solve

(a) $2^y = 8$

(1)

(b) $2^x \times 4^{x+1} = 8$

(4)

(Total for Question 3 is 5 marks)

Q4

Given

$$2^x \times 4^y = \frac{1}{2\sqrt{2}}$$

express y as a function of x

(Total for Question 4 is 3 marks)

Q5

Find, using algebra, all real solutions to the equation

(i) $16a^2 = 2\sqrt{a}$

(4)

(ii) $b^4 + 7b^2 - 18 = 0$

(4)

(Total for Question 5 is 8 marks)

Q6

The equation $kx^2 + 4kx + 3 = 0$, where k is a constant, has no real roots.

Prove that

$$0 \leqslant k < \frac{3}{4}$$

(Total for Question 6 is 4 marks)

Pearson

Q7

$$f(x) = x^2 + (k+3)x + k$$

where k is a real constant.

(a) Find the discriminant of $f(x)$ in terms of k.

(2)

(b) Show that the discriminant of $f(x)$ can be expressed in the form $(k + a)^2 + b$, where a and b are integers to be found.

(2)

(c) Show that, for all values of k, the equation $f(x) = 0$ has real roots.

(2)

(Total for Question 7 is 6 marks)

End of Questions

Silver Mark Scheme

Q1

Question Number	Scheme	Marks
(a)	$16^{\frac{1}{4}}=2$ or $\dfrac{1}{16^{\frac{1}{4}}}$ or better	M1
	$\left(16^{-\frac{1}{4}}=\right)\ \dfrac{1}{2}$ or 0.5 (ignore $\pm$)	A1
		(2)
(b)	$\left(2x^{-\frac{1}{4}}\right)^4=2^4x^{-\frac{4}{4}}$ or $\dfrac{2^4}{x^{\frac{4}{4}}}$ or equivalent	M1
	$x\left(2x^{-\frac{1}{4}}\right)^4=2^4$ or 16	A1 cao
		(2) **4**
	Notes	
(a)	M1 for a correct statement dealing with the $\frac{1}{4}$ or the $-$ power This may be awarded if 2 is seen or for reciprocal of their $16^{\frac{1}{4}}$ s.c ¼ is M1 A0 , also 2^{-1} is M1 A0 $\pm\frac{1}{2}$ is not penalised so M1 A1	
(b)	M1 for correct use of the power 4 on both the 2 and the x terms A1 for cancelling the x and simplifying to one of these two forms. Correct answers with no working get full marks	

Q2

Question Number	Scheme		Marks
	$\left\{\dfrac{2}{\sqrt{12}-\sqrt{8}}\right\} = \dfrac{2}{(\sqrt{12}-\sqrt{8})} \times \dfrac{(\sqrt{12}+\sqrt{8})}{(\sqrt{12}+\sqrt{8})}$	Writing this is sufficient for M1.	M1
	$= \dfrac{\{2(\sqrt{12}+\sqrt{8})\}}{12-8}$	For $12-8$. This mark can be implied.	A1
	$= \dfrac{2(2\sqrt{3}+2\sqrt{2})}{12-8}$		B1 B1
	$= \sqrt{3}+\sqrt{2}$		A1 cso
			5

Notes

M1: for a correct method to rationalise the denominator.

1st A1: $(\sqrt{12}-\sqrt{8})(\sqrt{12}+\sqrt{8}) \rightarrow 12-8$ or $(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2}) \rightarrow 3-2$

1st B1: for $\sqrt{12}=2\sqrt{3}$ or $\sqrt{48}=4\sqrt{3}$ seen or implied in candidate's working.

2nd B1: for $\sqrt{8}=2\sqrt{2}$ or $\sqrt{32}=4\sqrt{2}$ seen or implied in candidate's working.

2nd A1: for $\sqrt{3}+\sqrt{2}$. Note: $\dfrac{\sqrt{3}+\sqrt{2}}{1}$ as a final answer is A0.

Note: The first accuracy mark is dependent on the first method mark being awarded.

Note: $\frac{1}{2}\sqrt{12}+\frac{1}{2}\sqrt{8}=\sqrt{3}+\sqrt{2}$ with no intermediate working implies the B1B1 marks.

Note: $\sqrt{12}=\sqrt{4}\sqrt{3}$ or $\sqrt{8}=\sqrt{4}\sqrt{2}$ are not sufficient for the B1 marks.

Note: A candidate who writes down (by misread) $\sqrt{18}$ for $\sqrt{8}$ can potentially obtain M1A0B1B1A0, where the 2nd B1 will be awarded for $\sqrt{18}=3\sqrt{2}$ or $\sqrt{72}=6\sqrt{2}$

Note: The final accuracy mark is for a correct solution only.

Alternative 1 solution

$\left\{\dfrac{2}{\sqrt{12}-\sqrt{8}}\right\} = \dfrac{2}{(2\sqrt{3}-2\sqrt{2})}$ B1 B1

$= \dfrac{1}{(\sqrt{3}-\sqrt{2})} \times \dfrac{(\sqrt{3}+\sqrt{2})}{(\sqrt{3}+\sqrt{2})}$ M1

$= \dfrac{\{(\sqrt{3}+\sqrt{2})\}}{3-2}$ A1 for $3-2$

$= \sqrt{3}+\sqrt{2}$ A1

Please record the marks in the relevant places on the mark grid.

Alternative 2 solution

$\left\{\dfrac{2}{\sqrt{12}-\sqrt{8}}\right\} = \dfrac{2}{(2\sqrt{3}-2\sqrt{2})} = \dfrac{1}{(\sqrt{3}-\sqrt{2})} = \sqrt{3}+\sqrt{2}$, or $\dfrac{2}{(2\sqrt{3}-2\sqrt{2})} = \sqrt{3}+\sqrt{2}$

with no incorrect working seen is awarded M1A1B1B1A1.

Q3

Question Number	Scheme	Notes	Marks
(a)	$2^y = 8 \Rightarrow y = 3$	Cao (Can be implied i.e. by 2^3)	B1
	(Alternative: Takes logs base 2: $\log_2 2^y = \log_2 8 \Rightarrow y\log_2 2 = 3\log_2 2 \Rightarrow y = 3$)		
			(1)
(b)	$8 = 2^3$	Replaces 8 by 2^3 (May be implied)	M1
	$4^{x+1} = (2^2)^{x+1}$ or $(2^{x+1})^2$	Replaces 4 by 2^2 correctly.	M1
	$2^{3x+2} = 2^3 \Rightarrow 3x+2 = 3 \Rightarrow x = \frac{1}{3}$	M1: Adds their powers of 2 on the lhs and puts this equal to 3 leading to a solution for x.	M1A1
		A1: $x = \frac{1}{3}$ or $x = 0.\dot{3}$ or awrt 0.333	
			(4)
(b) Way 2	$4^{x+1} = 4 \times 4^x$	Obtains 4^{x+1} in terms of 4^x correctly	M1
	$2^x \times 4^x = 8^x$	Combines their 2^x and 4^x correctly	M1
	$4 \times 8^x = 8 \Rightarrow 8^x = 2 \Rightarrow x = \frac{1}{3}$	M1: Solves $8^x = k$ leading to a solution for x.	M1A1
		A1: $x = \frac{1}{3}$ or $x = 0.\dot{3}$ or awrt 0.333	
			[5]

Q4

Part	Working or answer an examiner might expect to see	Mark	Notes
	$2^x \times (2^2)^y = 2^{-\frac{3}{2}} \Rightarrow 2^{x+2y} = 2^{-\frac{3}{2}}$	M1	This mark is given for writing all terms in the same base and applying an index law
	$x + 2y = -\frac{3}{2}$	M1	This mark is given for writing an equation to link x and y
	$y = -\frac{1}{2}x - \frac{3}{4}$	A1	This mark is given for rearranging to find a correct expression of y as a function of x
			(Total 3 marks)

Q5

Question	Scheme		Marks	AOs
(i)	$16a^2 = 2\sqrt{a} \Rightarrow a^{\frac{3}{2}} = \frac{1}{8}$	$16a^2 - 2\sqrt{a} = 0$ $\Rightarrow 2a^{\frac{1}{2}}\left(8a^{\frac{3}{2}} - 1\right) = 0$ $\Rightarrow a^{\frac{3}{2}} = \frac{1}{8}$	M1	1.1b
	$\Rightarrow a = \left(\frac{1}{8}\right)^{\frac{2}{3}}$	$\Rightarrow a = \left(\frac{1}{8}\right)^{\frac{2}{3}}$	M1	1.1b
	$\Rightarrow a = \frac{1}{4}$	$\Rightarrow a = \frac{1}{4}$	A1	1.1b
	Deduces that $a = 0$ is a solution		B1	2.2a
			(4)	
(ii)	$b^4 + 7b^2 - 18 = 0 \Rightarrow \left(b^2 + 9\right)\left(b^2 - 2\right) = 0$		M1	1.1b
	$b^2 = -9, 2$		A1	1.1b
	$b^2 = k \Rightarrow b = \sqrt{k}, k > 0$		dM1	2.3
	$b = \sqrt{2}$, $-\sqrt{2}$ only		A1	1.1b
			(4)	
				(8 marks)

Notes

(i)

M1: Combines the two algebraic terms to reach $a^{\pm\frac{3}{2}} = C$ or equivalent such as $\left(\sqrt{a}\right)^3 = C$ $(C \neq 0)$

An alternative is via squaring and combining the algebraic terms to reach $a^{\pm 3} = k, k > 0$

E.g. $...a^4 = ...a \Rightarrow a^{\pm 3} = k$ or $...a^4 = ...a \Rightarrow ...a^4 - ...a = 0 \Rightarrow ...a\left(a^3 - ...\right) = 0 \Rightarrow a^3 = ...$

Allow for slips on coefficients.

M1: Undoes the indices correctly for their $a^{\frac{m}{n}} = C$ (So M0 M1 A0 is possible)
You may even see logs used.

A1: $a = \frac{1}{4}$ and no other solutions apart from 0 Accept exact equivalents Eg 0.25

B1: Deduces that $a = 0$ is a solution.

(ii)

M1: Attempts to solve as a quadratic equation in b^2
Accept $\left(b^2 + m\right)\left(b^2 + n\right) = 0$ with $mn = \pm 18$ or solutions via the use of the quadratic formula Also allow candidates to substitute in another variable, say $u = b^2$ and solve for u

A1: Correct solution. Allow for $b^2 = 2$ or $u = 2$ with no incorrect solution given.
Candidates can choose to omit the solution $b^2 = -9$ or $u = -9$ and so may not be seen

dM1: Finds at least one solution from their $b^2 = k \Rightarrow b = \sqrt{k}, k > 0$. Allow $b = 1.414$

A1: $b = \sqrt{2}$, $-\sqrt{2}$ only. The solution asks for real values so if $3i$ is given then score A0

Answers with minimal or no working:

In part **(i)**

- no working, just answer(s) with they can score the B1
- If they square and proceed to the quartic equation $256a^4 = 4a$ oe, and then write down the answers they can have access to all marks.

In part (ii)

- Accept for 4 marks $b^2 = 2 \Rightarrow b = \pm\sqrt{2}$
- No working, no marks.

Q6

Question	Scheme	Marks	AOs
	Realises that $k = 0$ will give no real roots as equation becomes 3 = 0 (proof by contradiction)	B1	3.1a
	(For $k \neq 0$) quadratic has no real roots provided $b^2 < 4ac$ so $16k^2 < 12k$	M1	2.4
	$4k(4k-3) < 0$ with attempt at solution	M1	1.1b
	So $0 < k < \frac{3}{4}$, which together with $k = 0$ gives $0 \leqslant k < \frac{3}{4}$ *	A1*	2.1
		(4 marks)	

Notes

B1 : Explains why $k = 0$ gives no real roots

M1 : Considers discriminant to give quadratic inequality – does not need the $k \neq 0$ for this mark

M1 : Attempts solution of quadratic inequality

A1*: Draws conclusion, which is a printed answer, with no errors (dependent on all three previous marks)

Q7

Question Number	Scheme	Marks
(a)	Discriminant: $b^2-4ac=(k+3)^2-4k$ or equivalent	M1 A1 (2)
(b)	$(k+3)^2-4k=k^2+2k+9=(k+1)^2+8$	M1 A1 (2)
(c)	For real roots, $b^2-4ac\geq 0$ or $b^2-4ac>0$ or $(k+1)^2+8>0$	M1
	$(k+1)^2\geq 0$ for all k, so $b^2-4ac>0$, so roots are real for all k (or equiv.)	A1 cso (2) **6**
	Notes (a) M1: attempt to find discriminant – substitution is required If formula b^2-4ac is seen at least 2 of a, b and c must be correct If formula b^2-4ac is **not** seen all 3 of a, b and c must be correct Use of b^2+4ac is M0 A1: correct unsimplified (b) M1: Attempt at completion of square (see earlier notes) A1: both correct (no ft for this mark) (c) M1: States condition as on scheme **or** attempts to explain that their $(k+1)^2+8$ is greater than 0 A1: The final mark (A1cso) requires $(k+1)^2\geq 0$ and conclusion. We will allow $(k+1)^2>0$ (or word positive) also allow $b^2-4ac\geq 0$ and conclusion.	

Gold Questions

Calculators may not be used

The total mark for this section is 33

Q1

Express 9^{3x+1} in the form 3^y, giving y in the form $ax + b$, where a and b are constants.

(Total for Question 1 is 2 marks)

Q2

The equation $x^2 + (k - 3)x + (3 - 2k) = 0$, where k is a constant, has two distinct real roots.
(a) Show that k satisfies

$$k^2 + 2k - 3 > 0$$

(3)

(b) Find the set of possible values of k.

(4)

(Total for Question 2 is 7 marks)

Q3

Given that the equation $2qx^2 + qx - 1 = 0$, where q is a constant, has no real roots,
(a) show that $q^2 + 8q < 0$.

(2)

(b) Hence find the set of possible values of q.

(3)

(Total for Question 3 is 5 marks)

Q4

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(i) Solve the equation

$$x\sqrt{2} - \sqrt{18} = x$$

writing the answer as a surd in simplest form.

(3)

(ii) Solve the equation

$$4^{3x-2} = \frac{1}{2\sqrt{2}}$$

(3)

(Total for Question 4 is 6 marks)

Q5

Given that $y = 2^x$,

(a) express 4^x in terms of y.

(1)

(b) Hence, or otherwise, solve

$$8(4^x) - 9(2^x) + 1 = 0$$

(4)

(Total for Question 5 is 5 marks)

Q6

$$f(x) = x^2 - 8x + 19$$

(a) Express f(x) in the form $(x + a)^2 + b$, where a and b are constants.

(2)

The curve C with equation $y = f(x)$ crosses the y-axis at the point P and has a minimum point at the point Q.

(b) Sketch the graph of C showing the coordinates of point P and the coordinates of point Q.

(3)

(c) Find the distance PQ, writing your answer as a simplified surd.

(3)

(Total for Question 6 is 8 marks)

End of Questions

Gold Mark Scheme

Q1

Question Number	Scheme	Notes	Marks
	9^{3x+1} = for example $3^{2(3x+1)}$ or $\left(3^2\right)^{3x+1}$ or $\left(3^{(3x+1)}\right)^2$ or $3^{3x+1}\times 3^{3x+1}$ or $(3\times 3)^{3x+1}$ or $3^2\times\left(3^2\right)^{3x}$ or $\left(9^{\frac{1}{2}}\right)^y$ or $9^{\frac{1}{2}y}$ or $y=2(3x+1)$	Expresses 9^{3x+1} correctly as a power of 3 or expresses 3^y correctly as a power of 9 or expresses y correctly in terms of x **(This mark is <u>not</u> for just $3^2=9$)**	M1
	$=3^{6x+2}$ or $y=6x+2$ or $a=6$, $b=2$	Cao (isw if necessary)	A1
	Providing there is no incorrect work, allow sight of $6x+2$ to score both marks Correct answer only implies both marks **Special case:** 3^{6x+1} **only** scores M1A0		
			[2]
	Alternative using logs		
	$9^{3x+1}=3^y\Rightarrow\log 9^{3x+1}=\log 3^y$		
	$(3x+1)\log 9=y\log 3$	Use power law correctly on both sides	M1
	$y=\dfrac{\log 9}{\log 3}(3x+1)$		
	$y=6x+2$	cao	A1
			2 marks

Q2

Question	Scheme	Marks	AOs
	Realises that $k=0$ will give no real roots as equation becomes 3 = 0 (proof by contradiction)	B1	3.1a
	(For $k\neq 0$) quadratic has no real roots provided $b^2<4ac$ so $16k^2<12k$	M1	2.4
	$4k(4k-3)<0$ with attempt at solution	M1	1.1b
	So $0<k<\frac{3}{4}$, which together with $k=0$ gives $0\leqslant k<\frac{3}{4}$ *	A1*	2.1
			(4 marks)

Notes

B1 : Explains why $k=0$ gives no real roots

M1 : Considers discriminant to give quadratic inequality – does not need the $k\neq 0$ for this mark

M1 : Attempts solution of quadratic inequality

A1*: Draws conclusion, which is a printed answer, with no errors (dependent on all three previous marks)

Q3

Question Number	Scheme	Marks
Q	b^2-4ac attempted, in terms of p. $(3p)^2-4p=0$ o.e. Attempt to solve for p e.g. $p(9p-4)=0$ Must potentially lead to $p=k,\ k\neq 0$ $p=\frac{4}{9}$ (Ignore $p=0$, if seen)	M1 A1 M1 A1cso [4]
	1st M1 for an attempt to substitute into b^2-4ac or $b^2=4ac$ with b or c correct Condone x's in one term only. This can be inside a square root as part of the quadratic formula for example. **Use of inequalities can score the M marks only** 1st A1 for any correct equation: $(3p)^2-4\times1\times p=0$ or better 2nd M1 for an attempt to factorize or solve their quadratic expression in p. Method must be sufficient to lead to their $p=\frac{4}{9}$. Accept factors or use of quadratic formula or $\left(p\pm\frac{2}{9}\right)^2=k^2$ (o.e. eg) $\left(3p\pm\frac{2}{3}\right)^2=k^2$ or equivalent work on <u>their</u> eqn. $9p^2=4p\Rightarrow\frac{9p^{\cancel{2}}}{\cancel{p}}=4$ which would lead to $9p=4$ is OK for this 2nd M1 ALT <u>Comparing coefficients</u> M1 for $(x+\alpha)^2=x^2+\alpha^2+2\alpha x$ and A1 for a correct equation eg $3p=2\sqrt{p}$ M1 for forming solving leading to $\sqrt{p}=\frac{2}{3}$ or better <u>Use of quadratic/discriminant formula (or any formula) Rule for awarding M mark</u> If the formula is quoted accept some correct substitution leading to a partially correct expression. If the formula is not quoted only award for a fully correct expression using their values.	

Q4

Question	Scheme	Marks	AOs
(i)	$x\sqrt{2}-\sqrt{18}=x \Rightarrow x\left(\sqrt{2}-1\right)=\sqrt{18} \Rightarrow x=\dfrac{\sqrt{18}}{\sqrt{2}-1}$	M1	1.1b
	$\Rightarrow x=\dfrac{\sqrt{18}}{\sqrt{2}-1}\times\dfrac{\sqrt{2}+1}{\sqrt{2}+1}$	dM1	3.1a
	$x=\dfrac{\sqrt{18}\left(\sqrt{2}+1\right)}{1}=6+3\sqrt{2}$	A1	1.1b
		(3)	
(ii)	$4^{3x-2}=\dfrac{1}{2\sqrt{2}} \Rightarrow 2^{6x-4}=2^{-\frac{3}{2}}$	M1	2.5
	$6x-4=-\dfrac{3}{2} \Rightarrow x=\ldots$	dM1	1.1b
	$x=\dfrac{5}{12}$	A1	1.1b
		(3)	
		(6 marks)	

Notes

(i)

M1: Combines the terms in x, factorises and divides to find x. Condone sign slips and ignore any attempts to simplify $\sqrt{18}$

Alternatively squares both sides $x\sqrt{2}-\sqrt{18}=x \Rightarrow 2x^2-12x+18=x^2$

dM1: Scored for a complete method to find x. In the main scheme it is for making x the subject and then multiplying both numerator and denominator by $\sqrt{2}+1$

In the alternative it is for squaring both sides to produce a 3TQ and then factorising their quadratic equation to find x. (usual rules apply for solving quadratics)

A1: $x=6+3\sqrt{2}$ only following a correct intermediate line. Allow $\dfrac{6+3\sqrt{2}}{1}$ as an intermediate line.

In the alternative method the $6-3\sqrt{2}$ must be discarded.

(ii)

M1: Uses correct mathematical notation and attempts to set both sides as powers of 2 or 4.

Eg $2^{ax+b}=2^{c}$ or $4^{dx+e}=4^{f}$ is sufficient for this mark.

Alternatively uses logs (base 2 or 4) to get a linear equation in x.

$4^{3x-2}=\dfrac{1}{2\sqrt{2}} \Rightarrow \log_2 4^{3x-2}=\log_2\dfrac{1}{2\sqrt{2}} \Rightarrow 2(3x-2)=\log_2\dfrac{1}{2\sqrt{2}}$.

Or $4^{3x-2}=\dfrac{1}{2\sqrt{2}} \Rightarrow 3x-2=\log_4\dfrac{1}{2\sqrt{2}}$

Or $4^{3x-2}=\dfrac{1}{2\sqrt{2}} \Rightarrow 4^{3x}=4\sqrt{2} \Rightarrow 3x=\log_4 4\sqrt{2}$

dM1: Scored for a complete method to find x.
Scored for setting the indices of 2 or 4 equal to each other and then solving to find x.
There must be an attempt on both sides.

You can condone slips for this mark Eg bracketing errors $4^{3x-2} = 2^{2\times 3x-2}$ or $\frac{1}{2\sqrt{2}} = 2^{-1+\frac{1}{2}}$

In the alternative method candidates cannot just write down the answer to the rhs.

So expect some justification. E.g. $\log_2 \frac{1}{2\sqrt{2}} = \log_2 2^{-\frac{3}{2}} = -\frac{3}{2}$

or $\log_4 \frac{1}{2\sqrt{2}} = \log_4 2^{-\frac{3}{2}} = -\frac{3}{2}\times\frac{1}{2}$ condoning slips as per main scheme

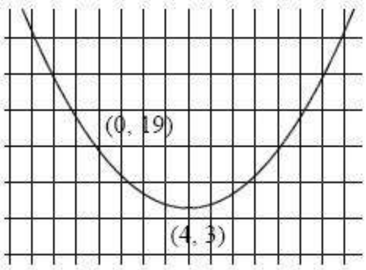
or $3x = \log_4 4\sqrt{2} \Rightarrow 3x = 1+\frac{1}{4}$

A1: $x = \frac{5}{12}$ with correct intermediate work

Q5

Question Number	Scheme		Marks
(a)	$(4^x =)y^2$	Allow y^2 or $y\times y$ or "y squared" "$4^x =$" not required	B1
	Must be seen in part (a)		
			(1)
(b)	$8y^2-9y+1=(8y-1)(y-1)=0 \Rightarrow y=...$ or $(8(2^x)-1)((2^x)-1)=0 \Rightarrow 2^x = ...$	For attempting to solve the given equation as a **3 term quadratic** in y or as a **3 term quadratic** in 2^x leading to a value of y or 2^x (Apply usual rules for solving the quadratic – see general guidance) Allow x (or any other letter) instead of y for this mark e.g. an attempt to solve $8x^2-9x+1=0$	M1
	$2^x (\text{or } y) = \frac{1}{8},\ 1$	Both correct answers of $\frac{1}{8}$ (oe) and 1 for 2^x or y or their letter but **<u>not x</u>** unless $2^x (\text{or } y)$ is implied later	A1
	$x=-3 \quad x=0$	M1: A correct attempt to find one **numerical value** of x from their $2^x (\text{or } y)$ **which must have come from a 3 term quadratic equation**. If logs are used then they must be evaluated. A1: Both $x=-3$ and/or $x=0$ May be implied by e.g. $2^{-3}=\frac{1}{8}$ and $2^0=1$ **and no extra values.**	M1A1
			(4)
			(5 marks)

Q6

Question Number	Scheme		Marks
(a)	$\mathrm{f}(x) = (x-4)^2 + 3$	M1: $\mathrm{f}(x) = (x \pm 4)^2 \pm \alpha,\ \alpha \neq 0$ (where α is a single number or a numerical expression $\neq 0$)	M1A1
		A1: Allow $\left(x + {}^{-}4\right)^2 + 3$ and ignore any spurious "= 0"	
	Allow a = -4, b = 3 to score both marks		
			(2)
(b)		B1: U shape anywhere even with no axes. Do not allow a "V" shape i.e. with an obvious vertex.	B1
		B1: P(0, 19). Allow (0, 19) or just 19 marked in the correct place as long as the curve (or straight line) passes through or touches here and allow (19, 0) as long as it is marked in the correct place. Correct coordinates may be seen in the body of the script as long as the curve (or straight line) passes through or touches here. If there is any ambiguity, the sketch has precedence. **(There must be a sketch to score this mark)**	B1
		B1: Q(4, 3). Correct coordinates that can be scored without a sketch but if a sketch is drawn then it must have a minimum in the first quadrant and no other turning points. May be seen in the body of the script. If there is any ambiguity, the sketch has precedence. Allow this mark if 4 is clearly marked on the x-axis below the minimum and 3 is marked clearly on the y-axis and corresponds to the minimum,	B1
			(3)

(c)	$PQ^2 = (0-4)^2 + (19-3)^2$	**Correct** use of Pythagoras' Theorem on 2 points of the form $(0, p)$ and (q, r) where $q \neq 0$ and $p \neq r$ with p, q and r numeric.	M1
	$PQ = \sqrt{4^2 + 16^2}$	Correct un-simplified numerical expression for PQ including the square root. **This must come from a correct *P* and *Q*.** Allow e.g $PQ = \sqrt{(0-4)^2 + (19-3)^2}$. Allow $\pm\sqrt{(0-4)^2 + (19-3)^2}$	A1
	$PQ = 4\sqrt{17}$	Cao and cso i.e. **This must come from a correct *P* and *Q*.**	A1
	Note that it is possible to obtain the correct value for PQ from (-4,3) and (0, 19) and e.g. (0, 13) and (4, -3) but the A marks in (c) can only be awarded for the correct P and Q.		
			(3)
			(8 marks)

Platinum Questions

Calculators may not be used

The total mark for this section is 18

1 A student was attempting to prove that $x = \frac{1}{2}$ is the only real root of

$$x^3 + \tfrac{3}{4}x - \tfrac{1}{2} = 0.$$

The attempted solution was as follows.

$$x^3 + \tfrac{3}{4}x = \tfrac{1}{2}$$

$\therefore$ $$x(x^2 + \tfrac{3}{4}) = \tfrac{1}{2}$$

$\therefore$ $$x = \tfrac{1}{2}$$

or $$x^2 + \tfrac{3}{4} = \tfrac{1}{2}$$

i.e. $$x^2 = -\tfrac{1}{4} \qquad \text{no solution}$$

$\therefore$ $$\text{only real root is } x = \tfrac{1}{2}$$

(a) Explain clearly the error in the above attempt.

(2)

(b) Give a correct proof that $x = \frac{1}{2}$ is the only real root of $x^3 + \frac{3}{4}x - \frac{1}{2} = 0$.

(3)

The equation

$$x^3 + \beta x - \alpha = 0 \qquad \text{(I)}$$

where α, β are real, $\alpha \neq 0$, has a real root at $x = \alpha$.

(c) Find and simplify an expression for β in terms of α and prove that α is the only real root provided $|\alpha| < 2$.

(6)

An examiner chooses a positive number α so that α is the only real root of equation (I) but the incorrect method used by the student produces 3 distinct real "roots".

(d) Find the range of possible values for α.

(7)

(Total for Question 1 is 19 marks)

End of Questions

Platinum Mark Scheme

Question Number	Scheme		Marks
7. (*a*)	$pq = \frac{1}{2} \not\Rightarrow p = \frac{1}{2}$ or $q = \frac{1}{2}$ (line 3)	identify; explain	B1; B1 (2)
(*b*)	$x^3 + \frac{3}{4}x - \frac{1}{2} = 0 \;\Rightarrow\; (x - \frac{1}{2})(x^2 + \frac{1}{2}x + 1) = 0$	attempt to divide	M1
		correct quadratic	A1
	i.e. $x = \frac{1}{2}$ or $x^2 + \frac{1}{2}x + 1 = 0$, discriminant $= (\frac{1}{2})^2 - 4$		M1
	$< 0 \;\therefore$ no real roots (so only root is $x = \frac{1}{2}$)		A1 cso (4)
(*c*)	$x = \alpha$ is a root $\Rightarrow \alpha^3 + \beta\alpha - \alpha = 0$, i.e. $\beta = 1 - \alpha^2$ $(\alpha \neq 0)$		M1, A1
	$x^3 + \beta x - \alpha \equiv (x - \alpha)[x^2 + \alpha x + 1]$		M1 [A1]
	Discriminant of $x^2 + \alpha x + 1$ is $\alpha^2 - 4$		M1
	$\therefore x = \alpha$ is the only real root if $\alpha^2 - 4 < 0$, i.e. $\lvert\alpha\rvert < 2$ (✱)		A1 cso (6)
(*d*)	Student's method: $x(x^2 + \beta) = \alpha$		
	$\Rightarrow x = \alpha$ or $x^2 + \beta = \alpha$		M1
	require $\alpha - \beta > 0$		
	$\alpha^2 + \alpha - 1 > 0$		
	cvs $\alpha = \dfrac{-1 \pm \sqrt{5}}{2}$	attempt cvs	M1
		2 correct cvs	A1
	$\therefore \dfrac{\sqrt{5}-1}{2} < \alpha < 2$ or $-2 < \alpha < -\dfrac{\sqrt{5}-1}{2}$		A1, A1 (7)
			(19 marks)

Pearson

(*a*)	STYLE INSIGHT & REASONING S marks For a novel or neat solution to any of questions 3—7. Apply once per question in up to 3 questions **S2** if solution is fully correct in principle and accuracy **S1** if principle is sound but includes a minor algebraic or numerical slip T mark **For a good and largely accurate attempt at the whole paper**	 S6 (S2 × 3) **T1** **(7 marks)**